Answer Key for Examples of Today's Homework Assignment Page 267-269

4. $\sum_{j=3}^{5} \frac{1}{j} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$

12. $\frac{2}{n} \sum_{i=1}^{n} \left[1 - \left(\frac{2i}{n} - 1\right)^2\right]$

16. $\sum_{i=1}^{15} (2i - 3) = 2\sum_{i=1}^{15} i - 3(15)$

$$= 2\left[\frac{15(16)}{2}\right] - 45 = 195$$

22. sum seq(x [^] 3 $-$ $2x$, x, 1, 15, 1) = 14,160 (*TI-82*)

$$\sum_{i=1}^{15} (i^3 - 2i) = \frac{(15)^2(15+1)^2}{4} - 2\,\frac{15(15+1)}{2}$$

$$= \frac{(15)^2(16)^2}{4} - 15(16) = 14{,}160$$

24. $S = [5 + 5 + 4 + 2](1) = 16$

$s = [4 + 4 + 2 + 0](1) = 10$

28. $S(8) = \left(\sqrt{\frac{1}{4}} + 2\right)\frac{1}{4} + \left(\sqrt{\frac{1}{2}} + 2\right)\frac{1}{4} + \left(\sqrt{\frac{3}{4}} + 2\right)\frac{1}{4} + \left(\sqrt{1} + 2\right)\frac{1}{4}$

$+ \left(\sqrt{\frac{5}{4}} + 2\right)\frac{1}{4} + \left(\sqrt{\frac{3}{2}} + 2\right)\frac{1}{4} + \left(\sqrt{\frac{7}{4}} + 2\right)\frac{1}{4} + \left(\sqrt{2} + 2\right)\frac{1}{4}$

$= \frac{1}{4}\left(16 + \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{5}}{2} + \frac{\sqrt{6}}{2} + \frac{\sqrt{7}}{2} + \sqrt{2}\right) \approx 6.038$

$s(8) = (0 + 2)\frac{1}{4} + \left(\sqrt{\frac{1}{4}} + 2\right)\frac{1}{4} + \left(\sqrt{\frac{1}{2}} + 2\right)\frac{1}{4} + \cdots + \left(\sqrt{\frac{7}{4}} + 2\right)\frac{1}{4} \approx 5.685$

34. $\lim_{n\to\infty}\left[\left(\frac{1}{n^2}\right)\frac{n(n+1)}{2}\right] = \frac{1}{2}\lim_{n\to\infty}\left[\frac{n^2+n}{n^2}\right] = \frac{1}{2}(1) = \frac{1}{2}$

36. $\sum_{j=1}^{n}\frac{4j+3}{n^2} = \frac{1}{n^2}\sum_{j=1}^{n}(4j+3) = \frac{1}{n^2}\left[\frac{4n(n+1)}{2} + 3n\right] = \frac{2n+5}{n} = S(n)$

$S(10) = \frac{25}{10} = 2.5$

$S(100) = 2.05$

$S(1000) = 2.005$

$S(10{,}000) = 2.0005$

40. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right) = \lim_{n\to\infty}\frac{4}{n^2}\sum_{i=1}^{n} i = \lim_{n\to\infty}\frac{4}{n^2}\left(\frac{n(n+1)}{2}\right) = \lim_{n\to\infty}\frac{4}{2}\left(1 + \frac{1}{n}\right) = 2$

50. $y = x^2 + 1$ on $[0, 3]$. $\left(\textbf{Note: } \Delta x = \dfrac{3 - 0}{n} = \dfrac{3}{n}\right)$

$$S(n) = \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\left(\frac{3}{n}\right) = \sum_{i=1}^{n}\left[\left(\frac{3i}{n}\right)^2 + 1\right]\left(\frac{3}{n}\right)$$

$$= \frac{27}{n^3}\sum_{i=1}^{n} i^2 + \frac{3}{n}\sum_{i=1}^{n} 1$$

$$= \frac{27}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \frac{3}{n}(n) = \frac{9}{2}\,\frac{2n^2 + 3n + 1}{n^2} + 3$$

$$\text{Area} = \lim_{n \to \infty} S(n) = \frac{9}{2}(2) + 3 = 12$$