Answer Key for Examples of Today's Homework   Pages 418-419

4.

$$\frac{dy}{dx} = 4 - y$$

$$\frac{dy}{4 - y} = dx$$

$$\int \frac{-1}{4 - y}\, dy = \int -dx$$

$$\ln|4 - y|\, dy = -x + C_1$$

$$4 - y = e^{-x + C_1} = Ce^{-x}$$

$$y = 4 - Ce^{-x}$$

12.

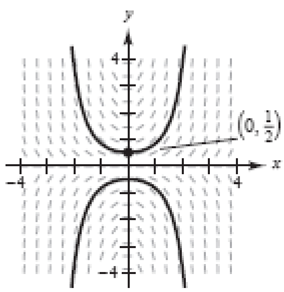
$$\frac{dP}{dt} = k(10 - t)$$

$$\int \frac{dP}{dt}\, dt = \int k(10 - t)\, dt$$

$$\int dP = -\frac{k}{2}(10 - t)^2 + C$$

$$P = -\frac{k}{2}(10 - t)^2 + C$$

16. (a)



(b) $\frac{dy}{dx} = xy,\ \left(0, \frac{1}{2}\right)$

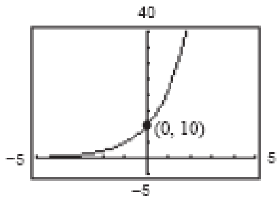
$$\frac{dy}{y} = x\, dx$$

$$\ln|y| = \frac{x^2}{2} + C$$

$$y = e^{x^2/2 + C} = C_1 e^{x^2/2}$$

$$\left(0, \frac{1}{2}\right): \frac{1}{2} = C_1 e^0 \Rightarrow C_1 = \frac{1}{2} \Rightarrow y = \frac{1}{2} e^{x^2/2}$$

20. $\frac{dy}{dt} = \frac{3}{4}y, \; (0, 10)$



$$\int \frac{dy}{y} = \int \frac{3}{4}\,dt$$

$$\ln y = \frac{3}{4}t + C_1$$

$$y = e^{(3/4)t + C_1}$$

$$= e^{C_1} e^{(3/4)t} = Ce^{3t/4}$$

$$10 = Ce^0 \implies C = 10$$

$$y = 10e^{3t/4}$$

22. $\frac{dN}{dt} = kN$

$N = Ce^{kt}$ (Theorem 5.16)

$(0, 250)$: $C = 250$

$(1, 400)$: $400 = 250e^k \implies k = \ln\frac{400}{250} = \ln\frac{8}{5}$

When $t = 4$, $N = 250e^{4\ln(8/5)} = 250e^{\ln(8/5)^4}$

$$= 250\left(\frac{8}{5}\right)^4 = \frac{8192}{5}.$$

26. $y = Ce^{kt}, \; (0, 4), \; \left(5, \frac{1}{2}\right)$

$C = 4$

$y = 4e^{kt}$

$\frac{1}{2} = 4e^{5k}$

$k = \frac{\ln(1/8)}{5} \approx -0.4159$

$y = 4e^{-0.4159t}$

34. Since the half-life is 1599 years,

$$\frac{1}{2} = 1e^{k(1599)}$$

$$k = \frac{1}{1599}\ln\left(\frac{1}{2}\right).$$

Since there are 1.5 g after 1000 years,

$$1.5 = Ce^{[\ln(1/2)/1599](1000)}$$

$$C \approx 2.314.$$

Hence, the initial quantity is approximately 2.314 g.

When $t = 10{,}000$, $y = 2.314e^{[\ln(1/2)/1599](10{,}000)}$

$$\approx 0.03 \text{ g}.$$

39. Since the half-life is 24,100 years,

$$\frac{1}{2} = 1e^{k(24{,}100)}$$

$$k = \frac{1}{24{,}100}\ln\left(\frac{1}{2}\right).$$

Since there are 2.1 grams after 1000 years,

$$2.1 = Ce^{[\ln(1/2)/24{,}100](1000)}$$

$$C \approx 2.161.$$

Thus, the initial quantity is approximately 2.161 g.

When $t = 10{,}000$, $y = 2.161e^{[\ln(1/2)/24{,}100](10{,}000)}$

$$\approx 1.62 \text{ g.}$$

Answer Key for Today's Homework Assignment   Pages 418-419

[6.2] p.418-419 #1,5,9,11,15,21,25,29,33,37,42

1. $\frac{dy}{dx} = x + 2$

$$y = \int (x + 2)\, dx = \frac{x^2}{2} + 2x + C$$

5. $y' = \frac{5x}{y}$

$$yy' = 5x$$

$$\int yy'\, dx = \int 5x\, dx$$

$$\int y\, dy = \int 5x\, dx$$

$$\frac{1}{2}y^2 = \frac{5}{2}x^2 + C_1$$

$$y^2 - 5x^2 = C$$

9. $(1 + x^2)y' - 2xy = 0$

$$y' = \frac{2xy}{1 + x^2}$$

$$\frac{y'}{y} = \frac{2x}{1 + x^2}$$

$$\int \frac{y'}{y}\, dx = \int \frac{2x}{1 + x^2}\, dx$$

$$\int \frac{dy}{y} = \int \frac{2x}{1 + x^2}\, dx$$

$$\ln|y| = \ln(1 + x^2) + C_1$$

$$\ln|y| = \ln(1 + x^2) + \ln C$$

$$\ln|y| = \ln[C(1 + x^2)]$$

$$y = C(1 + x^2)$$

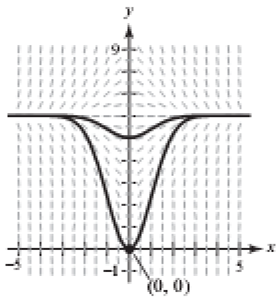
11. $\frac{dQ}{dt} = \frac{k}{t^2}$

$$\int \frac{dQ}{dt}\, dt = \int \frac{k}{t^2}\, dt$$

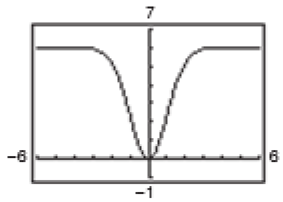
$$\int dQ = -\frac{k}{t} + C$$

$$Q = -\frac{k}{t} + C$$

15. (a)



(b)
$$\frac{dy}{dx} = x(6 - y), \quad (0, 0)$$
$$\frac{dy}{y-6} = -x\,dx$$
$$\ln|y - 6| = \frac{-x^2}{2} + C$$
$$y - 6 = e^{-x^2/2+C} = C_1e^{-x^2/2}$$
$$y = 6 + C_1e^{-x^2/2}$$
$$(0, 0)\text{: } 0 = 6 + C_1 \Rightarrow C_1 = -6 \Rightarrow y = 6 - 6e^{-x^2/2}$$



21. $\frac{dy}{dx} = ky$

$y = Ce^{kx}$ (Theorem 5.16)

$(0, 4)$: $4 = Ce^0 = C$

$(3, 10)$: $10 = 4e^{3k} \Rightarrow k = \frac{1}{3}\ln\left(\frac{5}{2}\right)$

When $x = 6$, $y = 4e^{1/3\,\ln(5/2)(6)} = 4e^{\ln(5/2)^2}$

$$= 4\left(\frac{5}{2}\right)^2 = 25.$$

25. $y = Ce^{kt}$, $\left(0, \frac{1}{2}\right)$, $(5, 5)$

$$C = \frac{1}{2}$$
$$y = \frac{1}{2}e^{kt}$$
$$5 = \frac{1}{2}e^{5k}$$
$$k = \frac{\ln 10}{5}$$
$$y = \frac{1}{2}e^{(\ln 10/5)t} = \frac{1}{2}(10^{t/5}) \text{ or } y \approx \frac{1}{2}e^{0.4605t}$$

29. In the model $y = Ce^{kt}$, C represents the initial value of y (when $t = 0$). k is the proportionality constant.

33. Since the initial quantity is 10 grams,

$$y = 10e^{kt}.$$

Since the half-life is 1599 years,

$$5 = 10e^{k(1599)}$$

$$k = \frac{1}{1599}\ln\left(\frac{1}{2}\right).$$

Thus, $y = 10e^{[\ln(1/2)/1599]t}$.

When $t = 1000$, $y = 10e^{[\ln(1/2)/1599](1000)} \approx 6.48$ g.

When $t = 10{,}000$, $y \approx 0.13$ g.

37. Since the initial quantity is 5 grams, $C = 5$.

Since the half-life is 5715 years,

$$2.5 = 5e^{k(5715)}$$

$$k = \frac{1}{5715}\ln\left(\frac{1}{2}\right).$$

When $t = 1000$ years, $y = 5e^{[\ln(1/2)/5715](1000)}$

$$\approx 4.43 \text{ g.}$$

When $t = 10{,}000$ years, $y = 5e^{[\ln(1/2)/5715](10{,}000)}$

$$\approx 1.49 \text{ g.}$$

42.

$$y = Ce^{kt}$$

$$\frac{1}{2}C = Ce^{k(5715)}$$

$$k = \frac{1}{5715}\ln\left(\frac{1}{2}\right)$$

$$0.15C = Ce^{[\ln(1/2)/5715]t}$$

$$\ln(0.15) = \frac{\ln\left(\frac{1}{2}\right)t}{5715}$$

$$t \approx 15{,}641.8 \text{ years}$$