

Answer Key for Examples of Today's Homework Page 409

4. Differential Equation: $\frac{dy}{dx} = \frac{xy}{y^2 - 1}$

Solution: $y^2 - 2 \ln y = x^2$

Check: $2yy' - \frac{2}{y}y' = 2x$

$$\left(y - \frac{1}{y}\right)y' = x$$

$$y' = \frac{x}{y - \frac{1}{y}}$$

$$y' = \frac{xy}{y^2 - 1}$$

10. $y = \frac{1}{2}x^2 - 4 \cos x + 2$

$$y' = x + 4 \sin x$$

Differential Equation:

$$y' = x + 4 \sin x$$

Initial condition:

$$y(0) = \frac{1}{2}(0)^2 - 4 \cos(0) + 2 = -4 + 2 = -2$$

16. $y = 5 \ln x$

$$y^{(4)} = -\frac{30}{x^4}$$

$$y^{(4)} - 16y = -\frac{30}{x^4} - 80 \ln x \neq 0,$$

No

20. $y = x^2 e^x$, $y' = x^2 e^x + 2x e^x = e^x(x^2 + 2x)$

$$xy' - 2y = x(e^x(x^2 + 2x)) - 2(x^2 e^x) = x^3 e^x,$$

Yes

28. $2x^2 - y^2 = C$ passes through (3, 4).

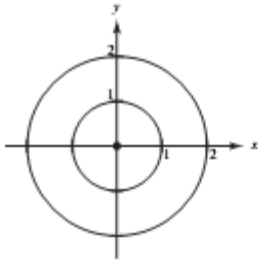
$$2(9) - 16 = C \Rightarrow C = 2$$

Particular solution: $2x^2 - y^2 = 2$

30. Differential equation: $yy' + x = 0$

General solution: $x^2 + y^2 = C$

Particular solutions: $C = 0$, Point $C = 1$, $C = 4$, Circles



34. Differential equation: $xy'' + y' = 0$

General solution: $y = C_1 + C_2 \ln x$

$$y' = C_2 \left(\frac{1}{x} \right), y'' = -C_2 \left(\frac{1}{x^2} \right)$$

$$xy'' + y' = x \left(-C_2 \frac{1}{x^2} \right) + C_2 \frac{1}{x} = 0$$