Answer Key for Today's Homework Assignment Pages 409-410

[6.1] p.409-410 #1-35 odd

1. Differential equation: $y' = 4y$

Solution: $y = Ce^{4x}$

Check: $y' = 4Ce^{4x} = 4y$

3. Differential equation: $y' = \dfrac{2xy}{x^2 - y^2}$

Solution: $x^2 + y^2 = Cy$

Check: $2x + 2yy' = Cy'$

$$y' = \frac{-2x}{(2y - C)}$$

$$y' = \frac{-2xy}{2y^2 - Cy}$$

$$= \frac{-2xy}{2y^2 - (x^2 + y^2)}$$

$$= \frac{-2xy}{y^2 - x^2}$$

$$= \frac{2xy}{x^2 - y^2}$$

5. Differential equation: $y'' + y = 0$

Solution: $y = C_1 \cos x + C_2 \sin x$

Check: $y' = -C_1 \sin x + C_2 \cos x$

$$y'' = -C_1 \cos x - C_2 \sin x$$

$$y'' + y = -C_1 \cos x - C_2 \sin x + C_1 \cos x + C_2 \sin x = 0$$

7. Differential Equation: $y'' + y = \tan x$

$$y = -\cos x \ln|\sec x + \tan x|$$

$$y' = (-\cos x)\frac{1}{\sec x + \tan x}(\sec x \cdot \tan x + \sec^2 x) + \sin x \ln|\sec x + \tan x|$$

$$= \frac{(-\cos x)}{\sec x + \tan x}(\sec x)(\tan x + \sec x) + \sin x \ln|\sec x + \tan x|$$

$$= -1 + \sin x \ln|\sec x + \tan x|$$

$$y'' = (\sin x)\frac{1}{\sec x + \tan x}(\sec x \cdot \tan x + \sec^2 x) + \cos x \ln|\sec x + \tan x|$$

$$= (\sin x)(\sec x) + \cos x \ln|\sec x + \tan x|$$

Substituting,

$$y'' + y = (\sin x)(\sec x) + \cos x \ln|\sec x + \tan x| - \cos x \ln|\sec x + \tan x|$$

$$= \tan x.$$

9. $y = \sin x \cos x - \cos^2 x$

$$y' = -\sin^2 x + \cos^2 x + 2\cos x \sin x$$

$$= -1 + 2\cos^2 x + \sin 2x$$

Differential Equation:

$$2y + y' = 2(\sin x \cos x - \cos^2 x) + (-1 + 2\cos^2 x + \sin 2x)$$

$$= 2\sin x \cos x - 1 + \sin 2x$$

$$= 2\sin 2x - 1$$

Initial condition:

$$y\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{4} - \cos^2\frac{\pi}{4} = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}\right)^2 = 0$$

11. $y = 6e^{-2x^2}$

$$y' = 6e^{-2x^2}(-4x) = -24xe^{-2x^2}$$

Differential Equation:

$$y' = -24xe^{-2x^2} = -4x(6e^{-2x^2}) = -4xy$$

Initial condition:

$$y(0) = 6e^{-2(0)^2} = 6e^0 = 6(1) = 6$$

In Exercises 13–18, the differential equation is $y^{(4)} - 16y = 0$.

13. $y = 3\cos x$

$y^{(4)} = 3\cos x$

$y^{(4)} - 16y = -45\cos x \neq 0,$

No

15. $y = e^{-2x}$

$y^{(4)} = 16e^{-2x}$

$y^{(4)} - 16y = 16e^{-2x} - 16e^{-2x} = 0,$

Yes

17. $y = C_1e^{2x} + C_2e^{-2x} + C_3\sin 2x + C_4\cos 2x$

$y^{(4)} = 16C_1e^{2x} + 16C_2e^{-2x} + 16C_3\sin 2x + 16C_4\cos 2x$

$y^{(4)} - 16y = 0,$

Yes

In 19–24, the differential equation is $xy' - 2y = x^3e^x$.

19. $y = x^2,\ y' = 2x$

$xy' - 2y = x(2x) - 2(x^2) = 0 \neq x^3e^x,$

No

21. $y = x^2(2 + e^x),\ \ y' = x^2(e^x) + 2x(2 + e^x)$

$xy' - 2y = x[x^2e^x + 2xe^x + 4x] - 2[x^2e^x + 2x^2] = x^3e^x,$

Yes

23. $y = \ln x, y' = \dfrac{1}{x}$

$$xy' - 2y = x\left(\frac{1}{x}\right) - 2\ln x \neq x^3e^x,$$

No

25. $y = Ce^{-x/2}$ passes through $(0, 3)$.

$3 = Ce^0 = C \implies C = 3$

Particular solution: $y = 3e^{-x/2}$

27. $y^2 = Cx^3$ passes through $(4, 4)$.

$16 = C(64) \implies C = \frac{1}{4}$

Particular solution: $y^2 = \frac{1}{4}x^3$ or $4y^2 = x^3$

29. Differential equation: $4yy' - x = 0$

General solution: $4y^2 - x^2 = C$

Particular solutions: $C = 0$, Two intersecting lines
$C = \pm 1,\ C = \pm 4$, Hyperbolas

31. Differential equation: $y' + 2y = 0$

General solution: $y = Ce^{-2x}$

$y' + 2y = C(-2)e^{-2x} + 2(Ce^{-2x}) = 0$

Initial condition: $y(0) = 3,\ 3 = Ce^0 = C$

Particular solution: $y = 3e^{-2x}$

33. Differential equation: $y'' + 9y = 0$

General solution: $y = C_1 \sin 3x + C_2 \cos 3x$

$y' = 3C_1 \cos 3x - 3C_2 \sin 3x,$

$y'' = -9C_1 \sin 3x - 9C_2 \cos 3x$

$$y'' + 9y = (-9C_1 \sin 3x - 9C_2 \cos 3x) + 9(C_1 \sin 3x + C_2 \cos 3x) = 0$$

35. Differential equation: $x^2y'' - 3xy' + 3y = 0$

General solution: $y = C_1x + C_2x^3$

$y' = C_1 + 3C_2x^2,\ y'' = 6C_2x$

$x^2y'' - 3xy' + 3y = x^2(6C_2x) - 3x(C_1 + 3C_2x^2) + 3(C_1x + C_2x^3) = 0$

Initial conditions: $y(2) = 0,\ y'(2) = 4$

$0 = 2C_1 + 8C_2$

$y' = C_1 + 3C_2x^2$

$4 = C_1 + 12C_2$

$$\left.\begin{aligned} C_1 + 4C_2 &= 0 \\ C_1 + 12C_2 &= 4 \end{aligned}\right\} \quad C_2 = \tfrac{1}{2},\ C_1 = -2$$

Particular solution: $y = -2x + \frac{1}{2}x^3$